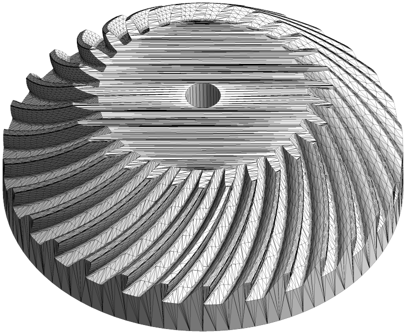
Intrinsic Surface Meshing Using Geodesic Distances

Tim GABRIEL – NEMESIS GMSH

Input Geometries

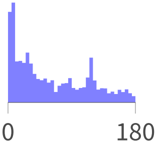
Triangulation as input



$\#f = 29164$

$\angle_{min} = 10^{-3\circ}$

$\#f_{area<10^{-5}} = 349$



Fine triangulation of the surface:

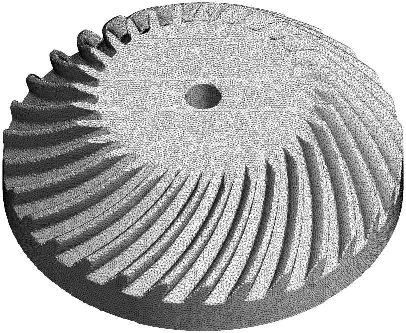
- Always available: CAD, Scan
- Exact representation of the geometry
- Correct topology

BUT

- Large amount of elements
- No guaranties on elements quality
- No guaranties on elements size/degeneracy

Input Geometries

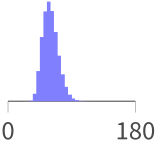
Remeshing of the geometry



$\#f = 133600$

$\angle_{\min} = 15.4^\circ$

$\#f_{area<10^{-5}} = 0$



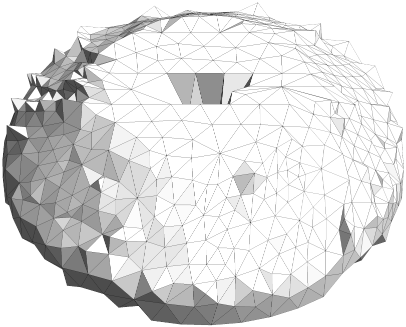
Remesh using a parametrization of the surface

- Simple linear element mesh
- Provide quality elements
- Ensures non degenerate elements
- Not exact geometry but close enough

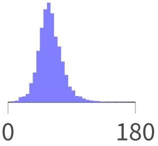
Still a large amount of elements

Input Geometries

Remeshing of the geometry for large elements



$\#f = 1688$
$\angle_{min} = 6.5°$
$\#f_{area<10^{-5}} = 0$



Remesh using a parametrization of the surface

- Simple linear element mesh
- Provide quality elements
- Ensures non degenerate elements
- Not exact geometry ~~but close enough~~

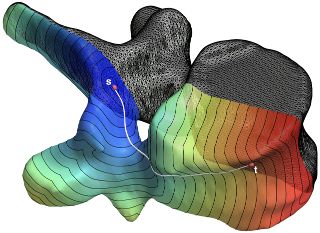
To get a small number of elements

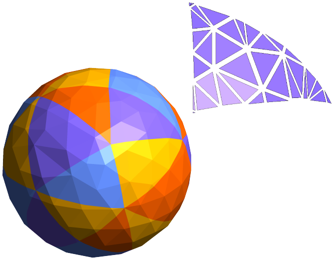
The Idea: use geodesic path instead of straight lines.

The Aim

Intrinsic Remeshing of Closed Surfaces

- Input: dense (and low quality) triangulation
- Tool: geodesic shortest path between points
- Benefits:
  - intrinsic lengths and angles
  - exact geometry
- Output: segmentation of geometry
- (Additional step: fit high-order elements)





Challenge 1: Fast computation of exact geodesic shortest paths on the triangulation.

Challenge 2: Implementing an intrinsic meshing algorithm.

Challenge 1 - Fast Exact Geodesics

Compute Geodesics

Unfold & Compare

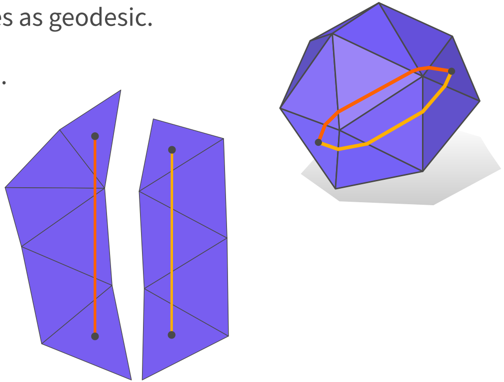
Always use the shortest path between two vertices as geodesic.

When unfolded, it correspond to planar distances.

To find exact shortest geodesic distances:

- Represent unfoldings
- Compute all the possible unfoldings
- Compare the distances

The MMP or ICH Algorithms.



[Mitchell, J. S., Mount, D. M., & Papadimitriou, C. H. (1987). The discrete geodesic problem]
[Chen, J., & Han, Y. (1990, May). Shortest paths on a polyhedron]
[Xin, S. Q., & Wang, G. J. (2009). Improving Chen and Han's algorithm on the discrete geodesic problem]

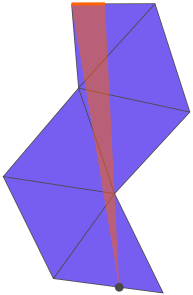
Compute Geodesics

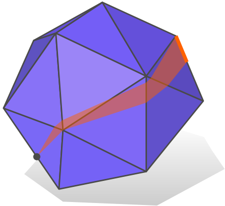
Unfold & Compare

MMP and ICH algorithms:

- Only distance to edges is needed
- An interval share the same unfolding.
- Only the position of the source in unfloded plane is needed.
- The shortest path can be found through backtracking towards the source.

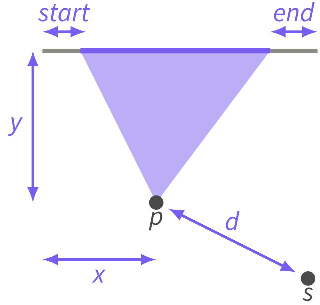
▶ Intervals as elementary objects.



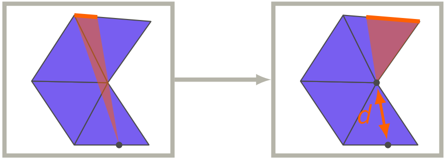


Compute Geodesics

Interval representation

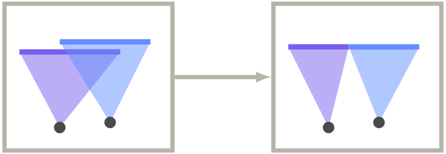


- 5 variables per interval.



Pseudo-sources (boundary/saddle points)

For MMP:



Interval intersection

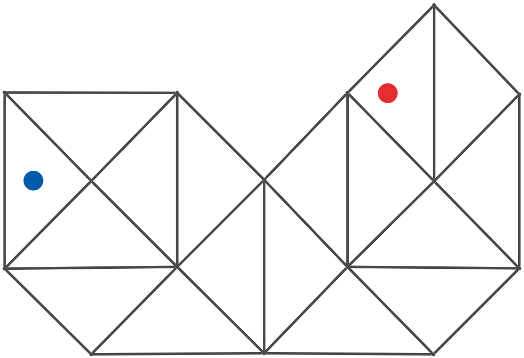
For ICH: filtering of intervals w.r.t. distances to vertices.

Compute Geodeiscs

Interval propagation

- Distance propagation from edges to edges.
- Ordered propagation to reach with minimum distance.
- Backtracking gives the shortest path.

  - Dijsktra like propagation



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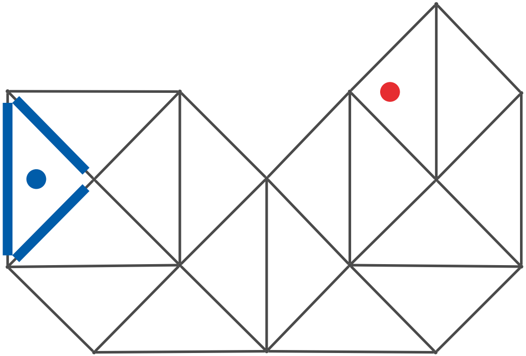

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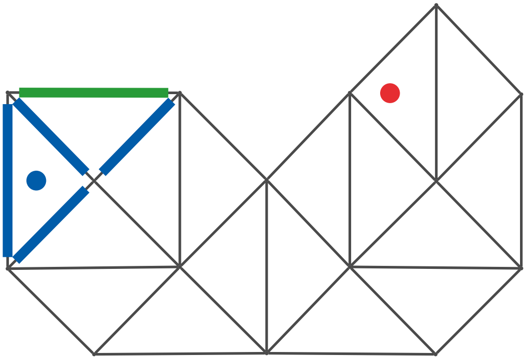

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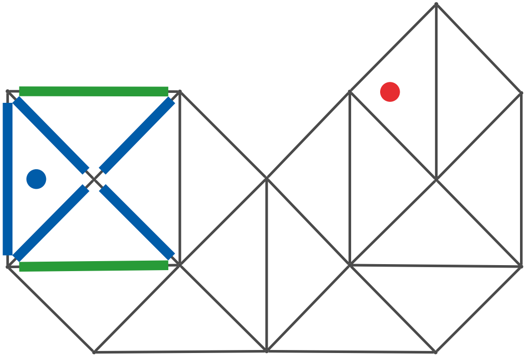

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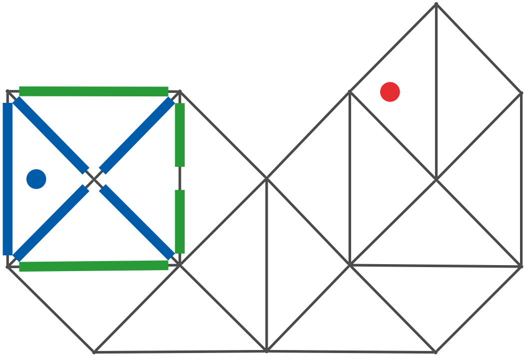

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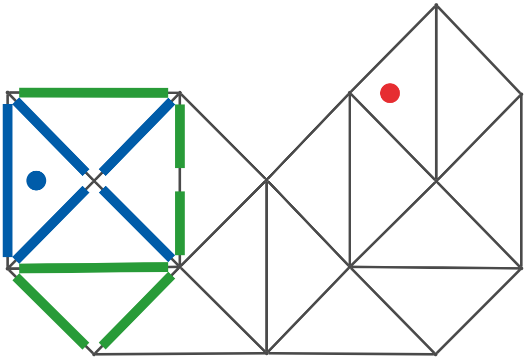

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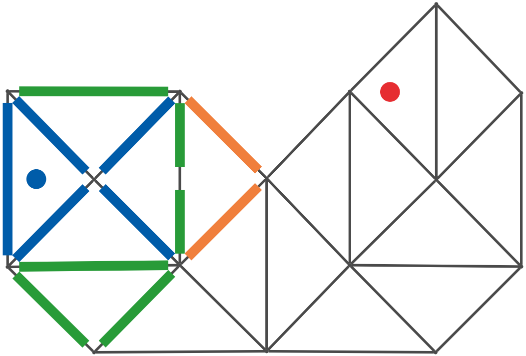

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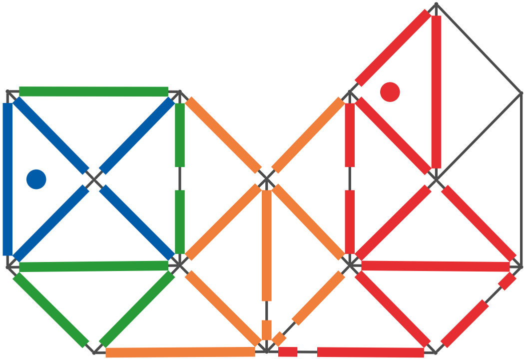

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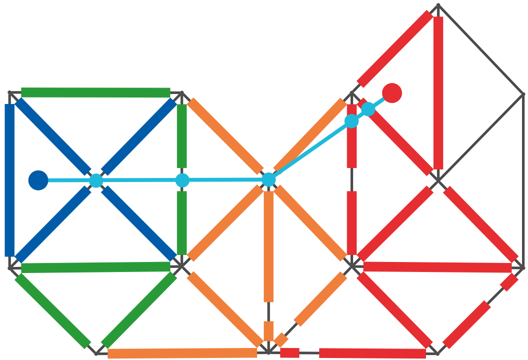


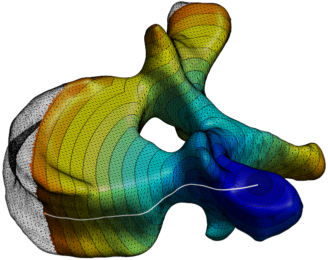
Compute Geodesics

Accelerate the computation

A^* search algorithm: propagate first the element with the lowest expected total distance.

Comparison values:

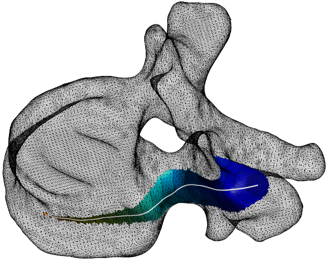
Classic MMP:	geodesicDistance(source, point)
A^* search:	geodesicDistance(source, point) + minDistance(point, destination)



Classic MMP

▶ 30 times faster and 20 times less intervals propagated (on a 160,000 faces geometry).

Note: a similar methods can find the point equidistant from three sources, if it exists.

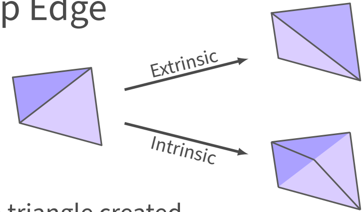


A^* search

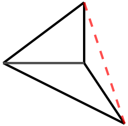
Challenge 2 - Intrinsic Meshing

Local Optimizations

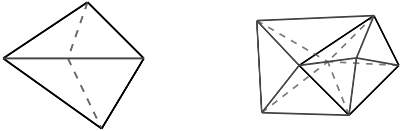
Swap Edge



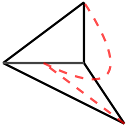
- No triangle created
- Criterion: empty circumcircles (Delaunay; optimize angles)
- No particular order required
- Common edges must intersect



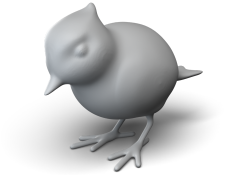
Split & Collapse Edge



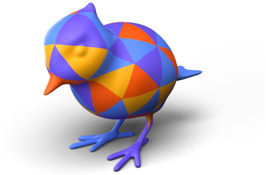
- Two triangles added/removed
- Criterion: target edge length
- Order matters:
  - Split largest first
  - Collapse smallest first
- New edges must be inside the cavity



First Pipeline

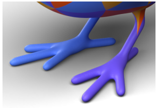


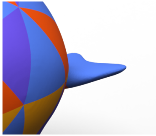
```
Swap edges
for i = 1 to N do
    Collapse edges
    Split edges
end for
```



- **Collapse edges:** remove small edges.
- **Split edges:** remove large edges.
- **Swap edges:** optimize triangle quality at the beginnig and after every other operation.

Problematic geodesic triangles

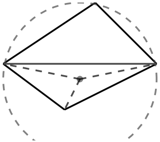




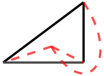
- Need "quality" triangles

Impose bounds on intrinsic angles.

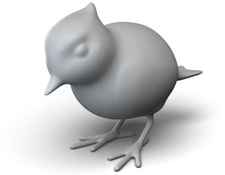
Split Triangle



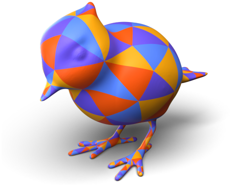
- Two triangles added
- Criterion: bounds on angles
- Not stricktly local:
  - Target triangle might not be split
  - Other triangle might be split
- No particular order is required.
- New edges must stay inside the cavity



Final Pipeline



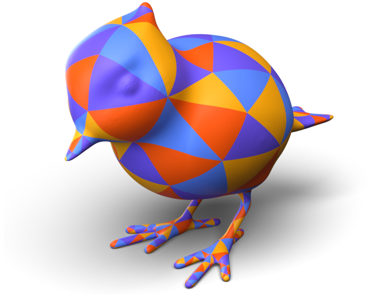
Swap edges
for i = 1 to N **do**
    Collapse edges*
    Split edges
    Split triangles
end for

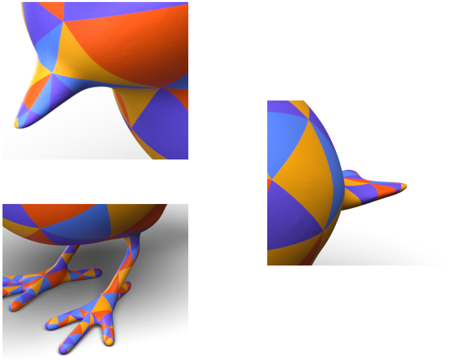


- **Collapse edges*:** remove small edges and while **ensuring angles in the bounds**.
- **Split edges:** remove large edges.
- **Split triangle:** split large triangles with angles out of the bounds.
- **Swap edges:** optimize triangle quality at the beginning and after every other operations.

Final Pipeline

Limit the angles of the triangles. (typically between 20° and 130°)

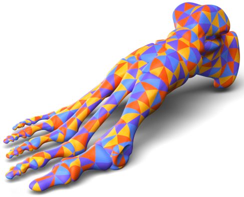




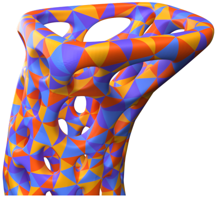
In order to converge: edges can not be collapsed w.r.t. angles bounds

Benchmarks

#Faces: 46,000
Time: 40 seconds
#Triangles: 670
Min angle: 13.9°
Max angle: 129.7°



#Faces: 196,000
Time: 55 seconds
#Triangles: 1184
Min angle: 20.4°
Max angle: 129.9°



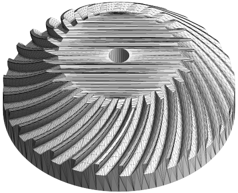
#Faces: 754,000
Time: 307 seconds
#Triangles: 3108
Min angle: 19.2°
Max angle: 129.6°

#Faces: 1,960,000
Time: 635 seconds
#Triangles: 2442
Min angle: 18.2°
Max angle: 135.9°

Example

Intrinsic remeshing of a gear

Initial mesh



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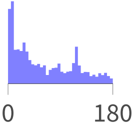
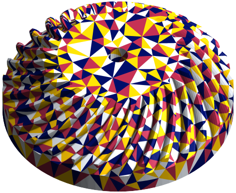


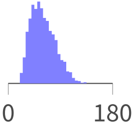
Intrinsic meshing
($\angle > 20^\circ$)



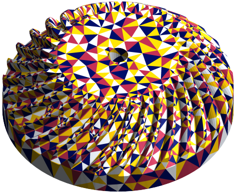
$\#\mathrm{f} = 1692$

$\angle_{\mathrm{min}} = 20.1^\circ$

$\#f_{area<10^{-5}} = 0$



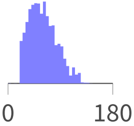
Intrinsic meshing
($\angle > 20^\circ$, $e_r < 1.1$)



$\#\mathrm{f} = 3954$

$\angle_{\mathrm{min}} = 3.3^\circ$

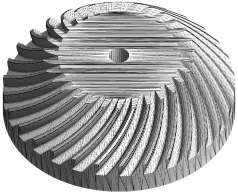
$\#f_{area<10^{-5}} = 0$



Example

Intrinsic remeshing of a gear

Initial mesh



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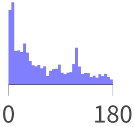
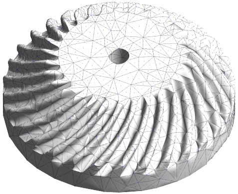


Intrinsic meshing
($\angle > 20^\circ$)



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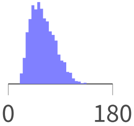
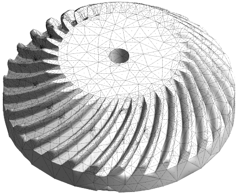


Intrinsic meshing
($\angle > 20^\circ$, $e_r < 1.1$)



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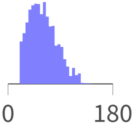
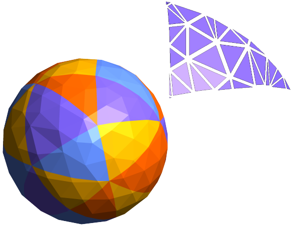
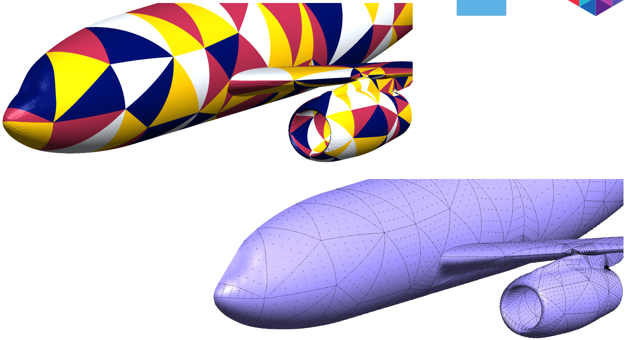


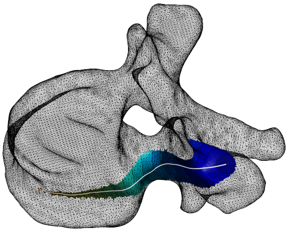
Perspectives



Intrinsic segmentation



Intrinsic remeshing and high order fitting



A* search

- Current bottleneck: time to compute geodesics.
- To explore: fit a way to exploit the raw segmentation.
- Next: open surfaces and volumes

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